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LETTERS TO THE EDITORS

THE PREDICTION OF THE TRANSPORT PROPERTIES OF A TURBULENT FLUID

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A PAPER [1] containing a new approach to the transport properties of turbulent flows is deserving of close attention. I have tried to clarify in my own mind what are the distinctive contributions of this paper, which features of the model are essential and which parts of the development are not strictly essential to the outcome. For example the size, shape and initial transverse velocity, V_0, of the entities cancel out when diffusivity ratios are taken and therefore do not influence the comparisons made with experimental data. Essential characteristics are that an entity moves off impulsively from its initial position, and thereafter is slowed down according to a laminar resistance law. It then follows from dimensional analysis that

$$\lambda^* = \frac{\rho R^2 V_0}{\mu} \times \left(\begin{matrix}\text{constant depending upon}\\ \text{shape of entity}\end{matrix}\right)$$

where R is a characteristic linear dimension of an entity. This is equivalent to the authors'

$$\lambda^* = \frac{\rho R^2 V_0}{\mu}\frac{2\psi}{9}.$$

Reference to an entity of ellipsoidal form is not an essential part of the development. The laminar resistance law is important in that it determines the time spent by the entity in traversing each element of its path of transverse length λ^*, and also the distribution along the path of the decay of its excess momentum corresponding to U_0.

I was puzzled for a time by the fact that it seemed possible to derive a turbulent shear stress, a rate of heat transport, etc., apparently without the frequency of creation of entities being specified. Despite the preamble to equation (13) one cannot define the contribution of a single entity to the shear stress; it is necessary to specify the rate at which entities are crossing unit area of the plane $y = 0$. What equation (13) in fact represents is the shear stress when the entire area of the plane $y = 0$ is being traversed by fluid with transverse velocity ΔV and velocity increment ΔU. (This is rather different from the picture of relatively widely spaced entities moving through fluid which has the mean motion). The next step is to determine the shear stress when entities with a range of ΔV, from V_0 to zero, and of ΔU, share the plane $y = 0$. This development in the paper leads to equation (21) and is worth close examination.

The required integral is an average value of $\Delta U \Delta V$ over unit area of the plane $y = 0$, i.e.

$$\int_0^1 \left(1 - \frac{\lambda}{\lambda^*}\right)^2 \ln\left(1 - \frac{\lambda}{\lambda^*}\right) \mathrm{d}a \qquad \text{(a)}$$

where $\mathrm{d}a$ is an element of the unit area. A relation is required between $\mathrm{d}a$ and $\mathrm{d}\lambda$.

According to the paper "the least prejudiced assignment of probability density to the value of (λ/λ^*) when an entity crosses the plane $y = 0$ is that the density is uniform for all values of (λ/λ^*) between 0 and 1". I do not find that an obvious proposition. It seems to me more natural to start with the postulate that entities are created at the same rate throughout the fluid. When they cross the plane $y = 0$ they occupy areas inversely proportional to ΔV in order to satisfy the continuity condition, i.e.

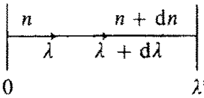
$$\mathrm{d}a \propto \frac{\mathrm{d}(\lambda/\lambda^*)}{1 - \lambda/\lambda^*}$$

and the above integral (a) becomes

$$\frac{\int_0^1 (1 - \lambda/\lambda^*) \ln(1 - \lambda/\lambda^*)\, \mathrm{d}\left(\frac{\lambda}{\lambda^*}\right)}{\int_0^1 \frac{\mathrm{d}(\lambda/\lambda^*)}{1 - \lambda/\lambda^*}}$$

However, a new difficulty immediately arises; the expression, and therefore the shear stress, is zero. The explanation is not hard to find; entities which take an infinite time to complete their motion over a transverse distance λ^* cannot produce a finite shear stress.

It would therefore seem essential to make provision in the model for entities to be eliminated after (on average) a finite time. An entity will from its moment of creation be liable to disappear in a new process of entity production. It would seem reasonable to assume that the probability of termination of any particular entity is uniformly distributed in time and the cumulative probability tends to unity as the time tends to infinity. Consider the decay of the entity flux $n(\lambda)$ with distance λ from the plane in which the entities are created at a steady rate n_0



Then $-\mathrm{d}n \propto$ entity flux $\times$ time of passage over element $\mathrm{d}\lambda$

$$\propto \frac{n}{v}\,\mathrm{d}\lambda$$

$$\propto \frac{n}{1-\dfrac{\lambda}{\lambda^*}}\,\mathrm{d}\lambda$$

i.e.

$$\frac{n}{n_0} = 1 - \frac{\lambda}{\lambda^*}$$

and the half-life of the entries is $(\lambda^*/V_0)\ln 2$, which is independent of V_0 since λ^* is proportional to V_0.

Returning to the expression (a) above we can now write

$$da \propto \frac{\text{entity flux } n(\lambda) \text{ from distance } \lambda}{\text{transverse velocity } (v)}\,\mathrm{d}\lambda$$

$$= \frac{1-\dfrac{\lambda}{\lambda^*}}{1-\dfrac{\lambda}{\lambda^*}}\,\mathrm{d}\left(\frac{\lambda}{\lambda^*}\right)$$

$$= \mathrm{d}\left(\frac{\lambda}{\lambda^*}\right).$$

The outcome is therefore precisely the authors' equation (21) but I, at least, am much clearer about its basis, and the process of entity termination has been introduced in a specific form.

On a point of detail it would appear that in taking the range $0 < \lambda < 1$ the authors have only considered entities traversing $y = 0$ in one direction [equation (15)]. Entities moving in the other direction would make an identical contribution to transport processes. However no results are affected since each group of entities would occupy only half of the plane $y = 0$.

HEAT TRANSPORT

Equations (41) and (43) are dimensionally incorrect. As defined in equation (42) ε_H must have the dimensions of thermal conductivity, not viscosity. The inconsistency has arisen between equations (40) and (41), and I believe that equation (41) should read

$$Q_H = \frac{K}{9} \langle \psi^1 N_R^2 \rangle \frac{P^2}{1 + 3P(\psi^1/\psi)} \frac{\mathrm{d}\langle T_f \rangle}{\mathrm{d}y}.$$

Note the square of Prandtl number in the numerator. In the paper the error in equation (41) is cancelled out when diffusivity ratios are taken as a result of some slight confusion over definitions in that ε is used in two distinct senses in the paper. The quantities ε_μ and ε_H as defined in equations (22) and (42) are not diffusivities but the turbulent viscosity μ_t and thermal conductivity K_t. ε_μ and ε_H should be reserved for the diffusivities as used later in equation (47) and Figs. 7, 9 and 10. Thus

$$\frac{\varepsilon_H}{\varepsilon_\mu} = \frac{K_t}{\rho c_p} \frac{\rho}{\mu_t}$$

and in this process the index of Prandtl number is reduced from 2 to 1 in agreement with equation (47).

I do not entirely accept the argument in section 5.2 that, following the assumption of uniformity of momentum within an entity, i.e. solid body motion, it was necessary for consistency to assume uniformity of temperature; it rather depends upon the Prandtl number. I strongly suspect that the correct justification is that, once the transient heat conduction process outside the entity has been included in the model, it is immaterial for the purposes of this paper whether internal heat conduction is also included or not. Similarly, as pointed out in the paper, once viscous effects outside an entity have been included, it is immaterial whether it is regarded as solid or fluid.

The physical basis of the curve in Fig. 7 is clear; at larger Prandtl numbers the entity moves a greater effective distance before giving up its thermal energy.

TURBULENT ENERGY TRANSPORT

I am afraid I have missed the argument of this section 5.1. The difficulty occurs between equations (24) and (25). Equation (24) gives the decay of kinetic energy (per unit mass) of an entity moving through a quiescent fluid. Is the kinetic energy of the entities related to the (undefined) turbulent energy? Is the energy $(U_0^2 + V_0^2 + W_0^2)$ of an entity at creation a measure of the turbulent energy density at its position of origin, which would seem to be consistent with equation (25)? My principal difficulty is to understand how, according to equation (25), the entity coming to rest at the end of its journey λ^* has completely adjusted to the turbulent energy level of its new surroundings. Perhaps equation (25) onwards is to be taken as a series of mathematical steps rather than a detailed argument with a physical basis?

PHYSICAL OR MATHEMATICAL MODEL?

Consider the problem of physically reconciling the assumptions made in the paper, which include:

(a) An entity moves off impulsively yet follows a resistance law appropriate to established flow.

(b) Heat transfer is calculated from a relation established for the case of no relative motion between the entity and its surroundings, which are also at constant temperature.

(c) Entities are treated as if widely spaced in a fluid whose motion is otherwise steady but elsewhere it is accepted that every plane perpendicular to the y-axis, and therefore the entire fluid, consists of entities in motion.

There is a natural desire, perhaps felt particularly strongly with the turbulence problem, for simple, consistent, comprehensible physical models, even at some risk of over-simplification, but I have sometimes felt that the feasibility of such models is not sufficiently distinguished from their desirability. Physical concepts are most valuable for constructing a length of track on which to start in motion a train of thought. The train tends to become airborne during the mathematical development section and when, much further on, it comes down on a convenient stretch of track we are inclined to ignore the fact that it was ever off the ground, and has perhaps, being a well-behaved train, now adjusted to a different gauge. At the end of our travel we ought to judge the correctness of our journey by the merits of our new surroundings, yet we tend also to retain the feeling that an additional recommendation is that it has followed a continuous physical line. My view is that the usefulness of the theory proposed by Mr. Tyldesley and Professor Silver must rest entirely on the correctness of its predictions, and that rather more evidence is required than diffusivity ratio comparisons, which are in any case difficult to measure accurately. It is general experience that alternative, and widely dissimilar, models can all show some initial success in predicting observed trends, provided that the basic physical laws of the situation have not been completely violated. I would rather doubt whether there is much more to be gained from the statistical mechanics approach to turbulent flow, but I will nevertheless await with interest further developments of this particular theory.

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REFERENCE

1. J. R. TYLDESLEY and R. S. SILVER, The prediction of the transport properties of a turbulent fluid, *Int. J. Heat Mass Transfer*, **11**, 1325–1340 (1968).

P.S. It has been brought to my attention by Mr. P. Bradshaw that a further immediate test of the proposed theory can be made. It is well known that in a jet and in pipe or boundary-layer flow outside the viscous sublayer the eddy diffusivity is practically independent of viscosity, whereas according to the theory in the paper ε_μ is inversely proportional to μ. This may well be the most serious objection to the theory as it stands.

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NOTE ON THE PREDICTION OF THE TRANSPORT PROPERTIES OF A TURBULENT FLUID

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TYLDESLEY and SILVER [1] have presented an interesting "rational description" of a turbulent fluid. Their model of coherent lumps of fluid, suddenly created, and surrounded by fluid having the properties of the mean flow, although it is hardly original, may lead to a number of useful qualitative conclusions. However when the quantitative analysis of [1] is applied to the turbulent flow of gas in a channel for example, there is a point at which the argument becomes fallacious. This arises from the failure of the treatment to evaluate the magnitudes of the length and velocity scales involved in the analysis, and it has at least three important repercussions.

Examination of the correlation coefficient of axial velocity fluctuations, R_y, as measured by Comte-Bellot [2], shows that it maintains a value greater than 0·5 for separations, y, of up to 0·1 of the channel half-width, h, over almost the whole of the flow passage (except for a region close to the wall). This indicates that an appropriate value for the radius of an "energy-containing entity" is $\langle R\rangle \sim 0{\cdot}1\ h$: moreover it is the energy-containing entities that make the most significant contribution to the transfer of momentum, which is described by the Reynolds stress in equation (13). That the authors [1] have overlooked this fact is shown by their comparison of equations (8) and (9), suggesting a correspondence between $\langle R^2\rangle^{\frac{1}{2}}$ and the microscale δ. The microscale, being a measure of turbulent velocity gradients and related to the rate of energy dissipation by viscosity, is a function of Reynolds number, and is considerably smaller